Efficient parameter estimation techniques for nonlinear MEMS resonators

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This presentation highlights theoretical and experimental results obtained within the framework of the CARAC-ATAC research project[1], led by GeePs, with C2N and ONERA. The aim of the project was to investigate techniques for performing fast characterization of high-Q, low-f_0 MEMS linear or nonlinear resonators, such as energy harvesters, resonant sensors, gyroscopes, etc. In particular, the time it takes to perform N measurements at different frequencies of the steady-state response of a resonator is typically one order of magnitude greater than $N \times Q/f_0$. This may be impractical in a variety of contexts. Furthermore, unless operating conditions are strictly controlled during this time, environmental drift may result in a large inaccuracy of the estimated parameters.

During the CARAC-ATAC project, we have developed techniques for performing such parameter estimation with excellent accuracy, and within a time of the order of Q/f_0. In their simplest installment, these techniques, whose roots lie in the domain of nonlinear vibration analysis[2], can routinely be used to estimate the parameters of resonators with polynomial nonlinear restoring, damping and driving forces, even in the presence of capacitive feedthrough. Thanks to the adopted formulation, the parameter estimation problem then boils down to a simple linear least-squares problem (nonlinear when feedthrough has to be accounted for). A typical result is represented in Fig. 1.

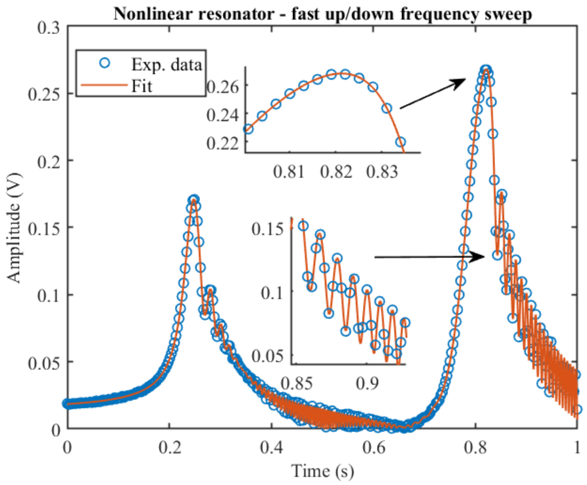

Fig. 1 – Experimental data (circles) resulting from a linearly-increasing ($t < 0{,}5s$) and decreasing fast frequency sweep (($t > 0{,}5s$)) and simulated data (line) after parameter estimation is performed.

The aim of the presentation is to give an illustrated overview of this project, some technical details relative to the implementation of the proposed techniques – so that they may be used by others, their pros and cons, and a comparison with other approaches, such as nonlinear ringdown[3].

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[3] P. M. Polunin et al., “Characterization of MEMS Resonator Nonlinearities Using the Ringdown Response”, Journal of Microelectromechanical Systems, vol. 25, pp. 297-303, 2016